Common Nouns as Variables and Modification by Adjectives (in a Variable-Free Semantics)¹

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0. The "Standard" System [Heim, Kratzer 1998]

Semantic types:

(1) e is the type of individuals.

t is the type of truth-values.

If σ and τ are semantic types, then $\langle \sigma, \tau \rangle$ ($\sigma \tau$ for short) is a semantic type.

 $\langle i, e \rangle$ is the type of assignment functions.

Semantic denotation domains:

(2) $D_e := D$ (the set of individuals).

 $D_t := \{0, 1\}$ (the set of truth-values).

For any semantic types σ and τ , $D_{\langle \sigma, \tau \rangle}$ is the set of all functions from D_{σ} to D_{τ} .

An assignment (function) is a partial function from |N to D.

(3) [.] is the interpretation function.

 $[\![\ldots]\!]^g$ means "... is interpreted relative to the assignment g"

Lexical items (with pronouns):

(4) $[Ann]^g = a$ (the individual named Ann)

• • •

 $[smoke]^g = [\lambda x: x \in D_e. x smokes]$ (square brackets and : ... may be omitted for brevity)²

...

 $^{^{1}}$ Please, note, that this handout shamelessly ignores any syntax whatsoever.

² Verbal agreement is also shamelessly ignored.

 $[hug]^g = \lambda y \lambda x. x hugs y$

. . .

 $[cellist]^g = \lambda x. x is a cellist$

. . .

 $[blonde]^g = \lambda x. x is blonde$

. . .

 $[she_6]^g = g(6)$ (i. e. that individual to which the assignment g maps the number 6)

$$[a_{\text{pred}}]^g = \lambda P_{\text{et}}. P^3$$

$$[\![be]\!]^g = \lambda P_{et}. P$$

Rules:

Functional Application (FA) (simplified)

If α and β are expressions and $\|\alpha\|^g$ is in the domain of $\|\beta\|^g$, then $\|\alpha\beta\|^g = \|\beta\|^g (\|\alpha\|^g)$.

(6) Predicate Modification (PM) (simplified)

If α and β are expressions and $[\![\alpha]\!]^g$ and $[\![\beta]\!]^g$ are of type et, then $[\![\alpha\beta]\!]^g = \lambda x \cdot [\![\beta]\!]^g(x) = [\![\alpha]\!]^g(x) = 1$.

Why are common nouns and adjectives semantically predicates?

Because they appear in the predicative position.

(7) $[Ann is blonde]^g = [is blonde]^g(a)^4 = [\lambda P. P](\lambda x. x is blonde)(a) = 1 iff a is blonde.$

³ Why would English ever need something of this kind?

Well, because apparently unlike adjectives common nouns can never combine with an NP of type e directly (or with a proper noun).

⁽i) With his new puppy *(a) husky, he's going to have to build a 6-foot fence around his yard. (adopted from [Jacobson 2014: 117])

⁽ii) With Mitka (a bit more) obedient, he might manage to graduate puppy class. [Ibid.: 114]

Whatever the syntax of nouns and adjectives, it must somehow specify that the latter sometimes can and the former can never combine with NPs directly.

⁴ Omitting some crucial steps.

[Ann is $a_{pred} \text{ cellist}$] $g = [is] g ([a_{pred} \text{ cellist}] g)(a) = [\lambda P. P]([\lambda P. P](\lambda x. x \text{ is a cellist}))(a) = 1 \text{ iff}$ a is a cellist.

Some more examples illustrate how PM and pronouns work.

- [Ann is a_{pred} blonde cellist][g] = [g] =
- [She₆ smokes] $^g = [smokes]^g(g(6)) = 1$ iff g(6) (the individual assigned to the index 6 by the assignment 6) smokes.

What in the world are these assignments and indices?

"[...] let us think of assignments as representing the contribution of the utterance context. The physical and psychological circumstances that prevail when an LF [logical form — S. M.] is processed will (if the utterance is felicitous) determine an assignment to all the free variables [non-bound pronouns — S. M.] occurring in this LF." [Heim, Kratzer 1998: 243]

In this system, the proposition expressed by a sentence is true iff the context determines an assignment under which the proposition is true.

1. Common Nouns as (Modally Non-rigid) Variables [Lasersohn 2019]

Peter Lasersohn pursues the hypothesis that common nouns are not predicates of individuals (type et), but rather individual variables (type e).

This has several important benefits:

- quantifiers are conservative without any stipulations
- bare NPs in articleless languages can compose without any additional hidden mechanisms or type shifters
- ...

Variables are expressions whose denotations are fixed directly by the assignment of values to variables. That is, α is a variable iff for all g (and all ways of filling in the three dots):

$$(4) \qquad \llbracket \alpha \rrbracket \cdots g = g(\alpha)$$

Variable *binding* is analyzed as the assignment of denotations relative to a given assignment g based on denotations relative to assignments which agree with g in what they assign to all variables other than the one being bound.¹ For example, we can define standard variable binding operators like \forall and \exists as in (5):

- (5) a. $[\![\forall \alpha \phi]\!]^{\dots,g} = 1$ iff $[\![\phi]\!]^{\dots,h} = 1$ for all h agreeing with g on all variables other than α .
 - b. $[\exists \alpha \phi] \dots g = 1$ iff $[\![\phi]\!] \dots h = 1$ for at least one h agreeing with g on all variables other than α .

We write 'TRACE(ξ) = ϵ ' to mean that ϵ marks the original position of ξ .

By the *antecedent* of a trace, let us mean the noun phrase of the quantifier phrase whose trace it is. That is, where δ is of category ((DP,Q)/NP) and κ is of category NP:

(7) ANTECEDENT(TRACE($\delta \kappa$)) = κ

independently of one another, for example. Let us write 'OCCURRENCE(α , A)' to mean that α is an occurrence of expression A, and 'CATEGORY(α , C)' to mean that α is an occurrence of an expression belonging to category C.

- g is an assignment of values to variables iff g is a function whose domain is included in {α | CATEGORY(α, NP) or CATEGORY(ANTECEDENT(α), NP)} such that:
 a. If α ∈ DOMAIN(g) and OCCURRENCE(α, professor) then g(α) is a professor;
- (13) α is of type σ iff for all assignments of values to variables g, $[\![\alpha]\!]^g \in \mathbf{D}_{\sigma}$.
- (14) For all assignments of values to variables g, h and all κ such that CATEGORY(κ , NP): $g \sim_{\kappa} h$ iff
 - a. There exists some x such that $h(\kappa) = x$, and for all ϵ such that ANTECEDENT(ϵ) = κ , $h(\epsilon) = x$, and
 - b. for all $v \in DOMAIN(g)$, if $v \neq \kappa$ and Antecedent $(v) \neq \kappa$, then h(v) = g(v).

Every "first pass":

(15) If OCCURRENCE(δ , every), CATEGORY(κ , NP) and CATEGORY(ϕ , TP), then for all assignments of values to variables $g: [\![\delta \kappa \phi]\!]^g = 1$ if $\forall h [\![g \sim_{\kappa} h \to [\![\phi]\!]^h = 1]$; 0 if $\exists h [\![g \sim_{\kappa} h \& [\![\phi]\!]^h = 0]$.

A less readable versions for further revisions:

(16) If OCCURRENCE(δ , every), CATEGORY(κ , NP) and CATEGORY(ϕ , TP), then for all assignments of values to variables $g: [\![\delta \kappa \phi]\!]g = 1$ if $\forall x \in \mathbf{D}_e[\exists h [g \sim_{\kappa} h \& h(\kappa) = x] \to \exists h [g \sim_{\kappa} h \& h(\kappa) = x \& [\![\phi]\!]^h = 1]];$ 0 if $\exists x \in \mathbf{D}_e \exists h [g \sim_{\kappa} h \& h(\kappa) = x \& [\![\phi]\!]^h = 0].$

Truth for now:

- (19) If CATEGORY(φ , TP) then:
 - a. φ is *true* iff $\llbracket \varphi \rrbracket^{\emptyset} = 1$; and
 - b. φ is *false* iff $\llbracket \varphi \rrbracket^{\emptyset} = 0$.

It may be helpful to work through an example. Let **Every professor e smiles** be an occurrence of the sentence *Every professor e smiles*. Suppose John, Mary, Bill and Susan are all the individuals there are. As before John and Mary are the professors, and Bill and Susan are the students. John, Bill and Mary all smile, but Susan does not.

- (20) a. **[Every professor e smiles]** $\emptyset = 1$ iff for $\forall x \in \mathbf{D}_e \ [\exists h [\emptyset \sim_{\mathbf{professor}} h \& h(\mathbf{professor}) = x] \rightarrow \exists h [\emptyset \sim_{\mathbf{professor}} h \& h(\mathbf{professor}) = x \& [[\mathbf{e smiles}]]^h = 1]].$
 - b. There is an h such that $\emptyset \sim_{\text{professor}} h$ and h(professor) = John, namely g_1 , where g_1 is that function with domain {professor, \mathbf{e} } such that $g_1(\text{professor}) = \text{John}$, $g_1(\mathbf{e}) = \text{John}$.
 - c. There is also an h such that $\emptyset \sim_{\mathbf{professor}} h$ and $h(\mathbf{professor}) = \mathbf{Mary}$, namely g_2 , where g_2 is that function with domain { $\mathbf{professor}$, \mathbf{e} } such that $g_2(\mathbf{professor}) = \mathbf{Mary}$ and $g_2(\mathbf{e}) = \mathbf{Mary}$.
 - d. Because John and Mary are the only two professors, they are the only individuals for which such assignments exist.
 - e. Since John and Mary both smile, $[e smiles]^{g_1} = 1$ and $[e smiles]^{g_2} = 1$.
 - f. Therefore **[Every professor e smiles]** $\emptyset = 1$.

Conservativity:

intuitive content of (21) is that A functions as a domain of quantification. That is, in ascertaining whether D(A, B), one need not consider those members of B which are not in A. Put differently (and more sloppily and English-specifically): in determining the truth value of a sentence of the form D N VP, one only need consider the N's: Which N's does VP apply to and which N's doesn't VP apply to? One never needs to consider the truth value that results from applying VP to something which isn't an N.

Informally, we can already see how conservativity falls out from the general approach to determiner quantification sketched in Section 2. Assignments of values to variables, in this approach, are functions from *common nouns* (and traces) to individuals. For any such function, the individual assigned to a given noun N is something which "is an N" — something which would be a member of the extension of N in a more conventional analysis. As one considers a class of assignments which differ at most in what they assign to N (and any trace anaphoric to N), therefore, one is effectively considering the things which are accurately described by the noun. As long as object-language quantification over individuals is analyzed in terms of metalanguage quantification over assignments of values to variables, and as long as determiner-noun combinations are interpreted by quantifying over those assignments which agree on all variables other than the noun with which the determiner combines (and its anaphors), conservativity is automatic.

In contrast, if we assume simply that determiners denote 2-place relations between sets (or functions of type $\langle\langle e, t \rangle, \langle\langle e, t \rangle, t \rangle\rangle$), nothing guarantees conservativity; it must be independently stipulated.⁶

(11) $[Every girl is a cellist] = \forall x.girl(x) ---> cellist(x)$

Indefinite article:

(54) If OCCURRENCE(δ, a) and CATEGORY(κ, NP), then for all $w \in W$ and all assignments g: $[\![\delta \kappa]\!]^{w,g} = g(\kappa, w)$

NPs with and without articles are thus of the same type.

Truth v2:

- (55) If $\varphi_1,...,\varphi_n$ is a text, then:
 - a. $\phi_1,...,\phi_n$ is *true relative to w* iff there is an assignment g such that $[\![\phi_1]\!]^{w,g} = 1,...,[\![\phi_n]\!]^{w,g} = 1$;
 - b. $\phi_1,...,\phi_n$ is *false relative to w* iff there is no assignment g such that $[\![\phi_1]\!]^{w,g} = 1,...,[\![\phi_n]\!]^{w,g} = 1;$
 - c. $\varphi_1,...,\varphi_n$ is *true* iff $\varphi_1,...,\varphi_n$ is true relative to w_0 ;
 - d. $\varphi_1,...,\varphi_n$ is *false* iff $\varphi_1,...,\varphi_n$ is false relative to w_0 .

An interesting feature of this analysis is that both definites and indefinites are of the same logical type as their nouns. This gives us a plausible explanation for why so many languages lack definite and/or indefinite articles: bare nouns are already of the right type (type e) to serve as arguments to a verb, even without an article. In the Latin sentence in (57), for example, the noun *librum* may serve directly as an argument to the verb $scrib\bar{o}$:

(57) Librum scrībō book.acc write.1sg.pres.indic "I write a/the book"

In contrast, in an analysis in which common nouns are of type $\langle e, t \rangle$, we must appeal to hidden determiners, or type-shifting operations, or something similar, in order to derive an expression of type e from the type $\langle e, t \rangle$ noun, so that it can fill the type e argument place of the verb. A very high proportion of languages do not have overt articles — very likely the majority. In light of this, it seems unnatural to have to treat nouns as unsuitable to serve as arguments to verbs unless some hidden structure or special operation is first applied to them. Even in languages like English, where articles are obligatory (at least with singular count nouns) in most stylistic registers, they are easily dropped in special styles such as recipes and instructions, headlines, tweets, etc. By treating nouns as variables of type e rather than as predicates of type e0, we have a reasonable explanation for why articles are so easily omitted.

Lasersohn further extends his system to treat:

- modally non-rigid variables, so that the denotations of common NPs vary with the worlds (unlike indexical pronouns)
- nominal complements such as friend of a professor
- relative clauses
 - (66) For all $w \in W$ and all assignments of values to variables g, if OCCURRENCE(ρ , Rel), then $[\![\rho]\!]^{w,g} = [\lambda p \colon p \in \mathbf{D}_t \& p = 1 \cdot [\lambda x \colon x \in \mathbf{D}_e \cdot x]\!]$
- the temperature paradox (Montague 1973)
- indexical pronouns

The system also automatically gives an analysis of:

- (at least some cases of) donkey anaphora
- the proportion problem
- bare plurals

Lasersohn also discusses some possible consequences and extensions:

- a unified collective conjunction
- raising just the quantifier
- eliminating (some major) part-of-speech distinctions

2. Extending the System

Lasersohn doesn't treat equative sentences such as Ann is a cellist and modification by adjectives as in A blonde cellist smokes, which are really some of the very basic cases that the system in [Heim, Kratzer 1998] can deal with.

Below I show that Lasersohn's system can be easily extended to treat these cases as well (without (multiple) stipulations) and as such is superior to the system in [Heim, Kratzer 1998].

2.1. From Adjectives to Equative Sentences and Back

The problem now:

$$\begin{split} \text{(12)} & & \text{[[cellist]]}^g = g(\text{cellist}) \\ & \dots \\ & \text{[[blonde]]}^g = \pmb{\lambda} x. \ x \ \text{is blonde} \end{split}$$

[13] [blonde cellist] $g = [\lambda x. x \text{ is blonde}](g(\text{cellist})) = 1 \text{ iff } g(\text{cellist}) \text{ is blonde}$ This is an expression of type t so it can't enter further composition. **Bad!** We can try to treat adjectives same as RCs above. Recall: phrase will be represented as [professor [Rel [who sees Susan]]]]. Suppose that in $w_{@}$, John and Mary are the professors, that John sees Susan and Mary does not see anyone. Suppose $g_1(\mathbf{professor}, w_{@}) = \mathbf{John}$:

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(69) a. g_1(\text{professor Rel who sees Susan}, w_@) = [\text{Rel who sees Susan}] w_@.g_1([\text{professor}] w_@.g_1)

b. = [\text{Rel who sees Susan}] w_@.g_1(g_1(\text{professor}, w_@))

c. = [\text{Rel who sees Susan}] w_@.g_1([\text{John}))

d. = [\lambda p : p \in D_t \& p = 1 . [\lambda x : x \in D_e . x]]([\text{who sees Susan}] w_@.g_1)([\text{John}))

e. = [\lambda p : p \in D_t \& p = 1 . [\lambda x : x \in D_e . x]]([\text{who}] w_@.g_1))([\text{John}))

f. = [\lambda p : p \in D_t \& p = 1 . [\lambda x : x \in D_e . x]]([\text{sees}] w_@.g_1(\text{Susan})(g_1(\text{who})))([\text{John}))

g. = [\lambda p : p \in D_t \& p = 1 . [\lambda x : x \in D_e . x]]([\text{sees}] w_@.g_1(\text{Susan})([\text{John})))

h. = [\lambda p : p \in D_t \& p = 1 . [\lambda x : x \in D_e . x]]([\text{sees}] w_@.g_1(\text{Susan})([\text{John})))

i. = [\lambda x : x \in D_e . x]([\text{John}))

j. = [\text{John}]
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The obvious solution (doesn't work):

- (14) $[blonde]^g = \lambda x: x \text{ is blonde. } x$
- (15) [blonde cellist] $^g = [\lambda x: x \text{ is blonde. } x](g(\text{cellist})) = g(\text{cellist}) \text{ (given g(cellist) is blonde)}^5$ Because it results in a wrong type in the predicative position:
- (16) [Ann is blonde]^g = [is blonde]^g(a)⁶ = [λ P. P](λ x: x is blonde. x)(a) = a (given a is blonde, undefined otherwise)

The semantics in (16) states that Ann is blonde means the individual a, which is obviously a bad result.

Then, let's treat adjectives as individual variables, too! (And give the verb be an equational semantics.)

⁵ Another problem of this solution is that the adjective's meaning is treated as a presupposition here (at least in the notation of [Heim, Kratzer 1998], while obviously it should be a part of at-issue content:

⁽iii) A: A loud man lives above.

B: No, he's just musically talented.

B's answer must negate the meaning of the adjective, which means that it must be a part of at-issue content (and not a presupposition).

⁶ Omitting some crucial steps.

- (17) [blonde]g = g(blonde) (i. e. the individual assigned to blonde by g) $[be]g = \lambda x \lambda y. \ x = y$
- [Ann is blonde]^g = [is blonde]^g(a)⁷ = [$\lambda x \lambda y$. x = y](g(blonde))(a) = 1 iff g(blonde) = a (i. e. Ann is blonde is true iff the individual assigned to blonde by g is a)

2.2. Prenominal Adjectives

But how should we interpret blonde cellist where both expressions are of type e?

(This reminds us of the same situation in the "standard" model, where both expressions are of type et. There it was resolved with PM, which is essentially set intersection and thus clearly doesn't apply to our case, since we're not dealing with sets.)

Assume a mereological sum operator \bigoplus that takes two individuals and returns a group formed from them. \bigoplus is idempotent, i. e. $\forall x [x \bigoplus x = x]$ (Krifka 1998).

- (19) Variable Modification (VM)

 If α and β are expressions and $[\![\alpha]\!]^g$ and $[\![\beta]\!]^g$ are of type e, then $[\![\alpha\beta]\!]^g = [\![\alpha]\!]^g \oplus [\![\alpha]\!]^g$, given $[\![\beta]\!]^g = [\![\alpha]\!]^g$, undefined otherwise.
- (20) [blonde cellist] $^g = g(blonde) \oplus g(cellist)$, given g(blonde) = g(cellist)Does blonde contribute to at-issue content, as it should? Apparently, yes.
- (21) Context: What do you see outside?

 [A blonde cellist smokes]^g = [smokes]^g(g(blonde) \oplus g(cellist)) = 1 iff g(blonde) \oplus g(cellist) smokes, given g(blonde) = g(cellist) (i. e. this sentence is true iff the mereological sum of the individual assigned to blonde by g and the individual assigned to cellist by g, given that the two are the same individual, smokes)
- (22) No, it's a brunette cellist.

 What does the answer in (22) negate?

⁷ Omitting some crucial steps.

It becomes clear if we restate (21) as (23).

(23) [A blonde cellist smokes] $^g = 1$ iff there exists an assignment g, such that $g(blonde) \oplus g(cellist)$ smokes, given g(blonde) = g(cellist)

Or:

[A blonde cellist smokes] $^g = 1$ iff the context determines an assignment g, such that $g(blonde) \oplus g(cellist)$ smokes, given g(blonde) = g(cellist)

The answer in (22) negates that there indeed exists such an assignment, but points out that there exists another assignment g', such that $[it's a brunette cellist]^{g'} = 1$.

BONUS. Common Nouns as Variables in a Variable-free Semantics

What in the world are these assignments and indices?

"[...] let us think of assignments as representing the contribution of the utterance context. The physical and psychological circumstances that prevail when an LF [logical form — S. M.] is processed will (if the utterance is felicitous) determine an assignment to all the free variables [non-bound pronouns — S. M.] occurring in this LF." [Heim, Kratzer 1998: 243]

In this system, the proposition expressed by a sentence is true iff the context determines an assignment (that determines the values of all free variables) under which the proposition is true.

As far as I can tell, if assignments are something like contexts, then indices are something like the referential intentions of the speaker. Thus, we need two formal devices to model how the referential intentions of a speaker in a context make the pronouns actually refer to some particular individuals.

Incidentally, we can achieve the same effect without any appeal to assignments, indices or even an LF⁸! That is the variable-free program advocated by Pauline Jacobson (e. g. [Jacobson 2014]).

⁸ An issue that requires some additional tinkering in an LF view. Is that Quantifier Raising (QR; movement of a quantifier phrase at LF to yield inverse scope) falsely predicts that the following sentence must have a meaning where the pronoun in the subject position is bound by the QR-ed quantifier phrase:

To recall, P. Jacobson argues that one can abandon the notion of assignments and indices (and (movement at) LF) to make for a simpler system accounting for the same range of facts.

<Sadly, I didn't have time to spell-out this section. I may attempt to do it online during the talk.>

3. Future Plans and Things to Deal with

How come different NPs scope differently dependent on the presence of the article? Can we derive it in his system? (Usually this follows from different types.)

- (24) Every boy read books. ∀ > booksIn the present system books must scope over every.
- (25) Ann has a false gun.

 Whatever happens with other adjectives?
- (26) Ann is a good cellist.
- $$\begin{split} \text{(27)} & \quad \text{[[very rude]]}^g = \pmb{\lambda} d \text{: } d > d_c \text{. } g(\text{rude}(d)) \\ & \quad \text{[[rude]]}^g = \pmb{\lambda} d \text{. } g(\text{rude}(d)) \\ & \quad \text{[[very]]}^g = \pmb{\lambda} P_{de} \pmb{\lambda} d \text{: } d > d_c \text{. } P(d) \in D_{\textit{de,de}} \end{split}$$

How does the noun contribute it's meaning in such cases?

(28) Four out of five balls are not under the sofa. #So it definitely must be under the sofa.

⁽iv) *His_i mother called no fourth grade boy_i (on the first day of school).

A variable-free (Direct Compositional) approach as developed by Jacobson does not fall prey to this (Weak Crossover effect) problem.

(29) I did book-reading. #They were interesting.

In (29) is a case a lot like pseudo-incorporation. This is usually analyzed in terms of the DP-NP divide. How can this system account for such cases?

(30) За боевые заслуги Васю произвели в капитаны почётного ордена. #В них он проходил два года ...

произвели'(v, капитан)

 \llbracket произвели $\rrbracket = \lambda \mathbf{A}_{ge}.\lambda \mathbf{x}.$ произвели' $(x,\,\mathbf{A})$

- (31) Вася всегда хотел стать капитаном. OK И он им стал. (СЮ: тут другое oh.)
- (32) $[the] = \lambda P: |P| = 1 \land P \cap D. P$

[The boy came] = $[\lambda z. [\lambda x. x came](z) = [\lambda y. y is a boy](z) = 1]$ (given $|[boy]| = 1 \land [boy]$ $\cap D$)

* EXISTENTIAL BINDING *

 $\exists z. [\lambda x. x \text{ came}](z) = [\lambda y. y \text{ is a boy}](z) = 1 \text{ (given } |[boy]] = 1 \land [boy] \cap D)$

[The boy pet the dog] = ???

If nouns are et, then we have to get some additional machinery to encode participant roles. (This is all due to SYu and Fedya.)

References

Heim, Kratzer 1998 —

Jacobson 2014 —

Krifka 1998 —

Lasersohn 2019 —

Montague 1973 —